Decision Analysis

- Problem Formulation
- Decision Making without Probabilities
- Decision Making with Probabilities

Problem Formulation

- A decision problem is characterized by decision alternatives, states of nature, and resulting payoffs.
- The decision alternatives are the different possible strategies the decision maker can employ.
- The states of nature refer to future events, not under the control of the decision maker, which may occur. States of nature should be defined so that they are mutually exclusive and collectively exhaustive.

Influence Diagrams

- An influence diagram is a graphical device showing the relationships among the decisions, the chance events, and the consequences.
- Squares or rectangles depict decision nodes.
- Circles or ovals depict chance nodes.
- Diamonds depict consequence nodes.
- Lines or arcs connecting the nodes show the direction of influence.

Payoff Tables

- The consequence resulting from a specific combination of a decision alternative and a state of nature is a payoff.
- A table showing payoffs for all combinations of decision alternatives and states of nature is a payoff table.
- Payoffs can be expressed in terms of profit, cost, time, distance or any other appropriate measure.

Decision Trees

- A decision tree is a chronological representation of the decision problem.
- Each decision tree has two types of nodes; round nodes correspond to the states of nature while square nodes correspond to the decision alternatives.
- The branches leaving each round node represent the different states of nature while the branches leaving each square node represent the different decision alternatives.
- At the end of each limb of a tree are the payoffs attained from the series of branches making up that limb.

Decision Making without Probabilities

- Three commonly used criteria for decision making when probability information regarding the likelihood of the states of nature is unavailable are:
  - the <u>optimistic</u> approach
  - the <u>conservative</u> approach
  - the <u>minimax regret</u> approach.

Optimistic Approach

- The optimistic approach would be used by an optimistic decision maker.
- The decision with the largest possible payoff is chosen.
- If the payoff table was in terms of costs, the decision with the lowest cost would be chosen.

Conservative Approach

- The conservative approach would be used by a conservative decision maker.
- For each decision the minimum payoff is listed and then the decision corresponding to the maximum of these minimum payoffs is selected. (Hence, the minimum possible payoff is maximized.)
- If the payoff was in terms of costs, the maximum costs would be determined for each decision and then the decision corresponding to the minimum of these maximum costs is selected. (Hence, the maximum possible cost is minimized.)

Minimax Regret Approach

- The minimax regret approach requires the construction of a regret table or an opportunity loss table.
- This is done by calculating for each state of nature the difference between each payoff and the largest payoff for that state of nature.
- Then, using this regret table, the maximum regret for each possible decision is listed.
- The decision chosen is the one corresponding to the minimum of the maximum regrets.

Example

Consider the following problem with three decision alternatives and three states of nature with the following payoff table representing profits:

		States of Nature		
		s_1	s_2	s_3
	d_1	4	4	-2
Decisions	d_2	0	3	-1
	d_3	1	5	-3

Example

- Optimistic Approach

  An optimistic decision maker would use the optimistic (maximax) approach. We choose the decision that has the largest single value in the payoff table.

Decision	Maximum Payoff
4	
3	
5	

d

Maximax decision

Maximax payoff

Example

- Conservative Approach

  A conservative decision maker would use the conservative (maximin) approach. List the minimum payoff for each decision. Choose the decision with the maximum of these minimum payoffs.

Decision	Minimum Payoff
-2	
d_2 -1	
d_3	-3

Maximin decision

Maximin payoff

Example

- Minimax Regret Approach

  For the minimax regret approach, first compute a regret table by subtracting each payoff in a column from the largest payoff in that column. In this example, in the first column subtract 4, 0, and 1 from 4; etc. The resulting regret table is:

	s_1	s_2	s_3
d_1	0	1	1
d_2	4	2	0
d_3	3	0	2

Example

- Minimax Regret Approach (continued)

  For each decision list the maximum regret. Choose the decision with the minimum of these values.

	Maximum
Decision	Regret
d_1	1
d_2	4
d_3	3

Minimax decision

Minimax regret

Decision Making with Probabilities

- Expected Value Approach
  - If probabilistic information regarding the states of nature is available, one may use the expected value (EV) approach.
  - Here the expected return for each decision is calculated by summing the products of the payoff under each state of nature and the probability of the respective state of nature occurring.
  - The decision yielding the best expected return is chosen.

Expected Value of a Decision Alternative

- The <u>expected value of a decision alternative</u> is the sum of weighted payoffs for the decision alternative.
- The expected value (EV) of decision alternative d_i is defined as:

$$\text{EV}(d_i) = \sum_{j=1}^{N} P(s_j)V_{ij}$$

where: N = the number of states of nature
$P(s_j)$ = the probability of state of nature s_j
V_{ij} = the payoff corresponding to decision alternative d_i and state of nature s_j

Example: Burger Prince

Burger Prince Restaurant is contemplating opening a new restaurant on Main Street. It has three different models, each with a different seating capacity. Burger Prince estimates that the average number of customers per hour will be 80, 100, or 120. The payoff table for the three models is on the next slide.

Example: Burger Prince

- Payoff Table

Average Number of Customers Per Hour

	$s_1 = 80$	$s_2 = 100$	$s_3 = 120$
Model A	$10,000	$15,000	$14,000
Model B	$ 8,000	$18,000	$12,000
Model C	$ 6,000	$16,000	$21,000

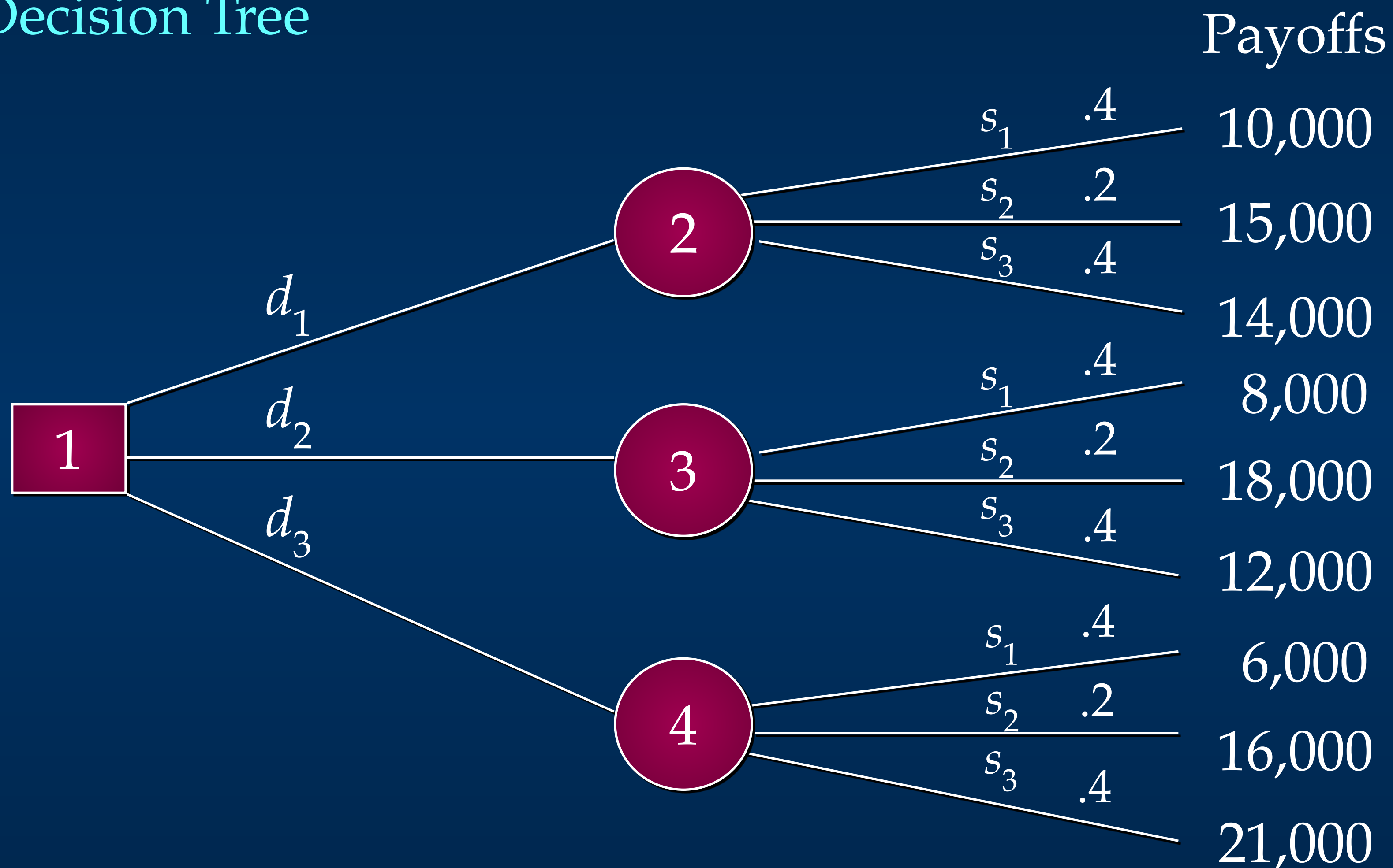
Example: Burger Prince

- Expected Value Approach

  Calculate the expected value for each decision. The decision tree on the next slide can assist in this calculation. Here d_1, d_2, d_3 represent the decision alternatives of models A, B, C, and s_1, s_2, s_3 represent the states of nature of 80, 100, and 120.

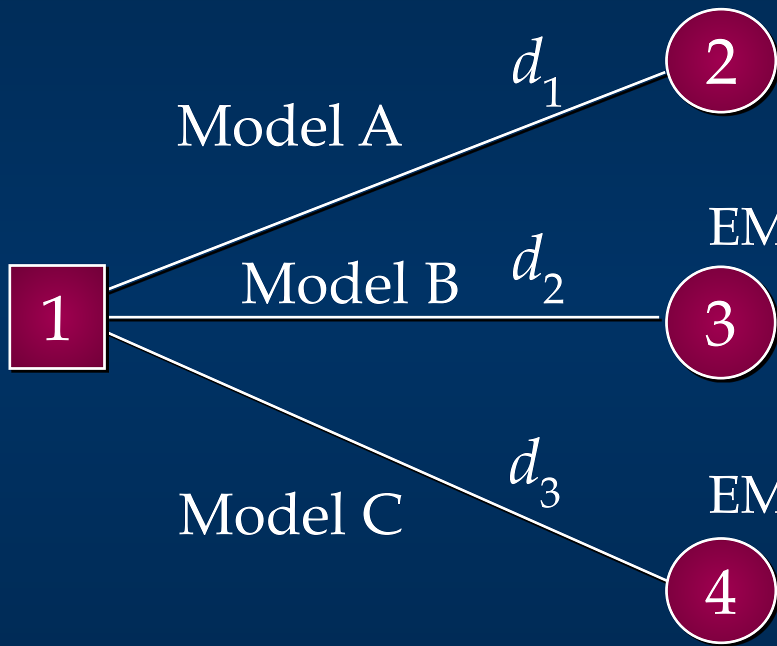
Example: Burger Prince

- Decision Tree



Example: Burger Prince

- Expected Value For Each Decision



EMV = .4(10,000) + .2(15,000) + .4(14,000)
= \$12,600

EMV = .4(8,000) + .2(18,000) + .4(12,000)
= \$11,600

EMV = .4(6,000) + .2(16,000) + .4(21,000)
= \$14,000

Choose the model with largest EV, Model C.

Expected Value of Perfect Information

- Frequently information is available which can improve the probability estimates for the states of nature.
- The expected value of perfect information (EVPI) is the increase in the expected profit that would result if one knew with certainty which state of nature would occur.
- The EVPI provides an upper bound on the expected value of any sample or survey information.

Expected Value of Perfect Information

- EVPI Calculation
  - Step 1:

    Determine the optimal return corresponding to each state of nature.
  - Step 2:

    Compute the expected value of these optimal returns.
  - Step 3:

    Subtract the EV of the optimal decision from the amount determined in step (2).

Example: Burger Prince

- Expected Value of Perfect Information

  Calculate the expected value for the optimum payoff for each state of nature and subtract the EV of the optimal decision.

EVPI= .4(10,000) + .2(18,000) + .4(21,000) - 14,000 = $2,000